To be presented on Oct 26 ’10

8.1. (a) $N$ identical fermions (spin 1/2) of mass $m$ move in the one-dimensional potential $V(x) = \frac{1}{2} m \omega^2 x^2$. Find the ground state energy for $N = 1$, $N = 2$, $N = 3$, $N = 4$ and $N = 5$.

(b) Same problem when the particles are identical bosons.

(c) Additionally, a magnetic field $B$ is applied. The strength of the field is such that $\mu B = \hbar \omega$, where $\mu$ is the magnetic moment of the particle. Find the ground state energies as in (a).

Solution to 8.1.

(a) The single particle oscillator energy levels: $E_n = \hbar \omega \left( n + \frac{1}{2} \right)$. For a given $n$, there are two possible spin orientation so that there are two quantum states with the same energy (2-fold degeneracy). In accordance with the Pauli principle (fermions), any quantum state can occupied only once. Therefore, the energy level $E_n$ can be occupied by no more than two fermions. In the ground state, fermions sit on the lowest possible levels, compatible with the Pauli principle:

\[ E_{N=1} = E_{n=0} = \frac{1}{2} \hbar \omega \]  
\[ E_{N=2} = 2E_{n=0} = \hbar \omega \]  
\[ E_{N=3} = 2E_{n=0} + E_{n=1} = \frac{5}{2} \hbar \omega \]  
\[ E_{N=4} = 2E_{n=0} + 2E_{n=1} = 4 \hbar \omega \]  
\[ E_{N=5} = 2E_{n=0} + 2E_{n=1} + E_{n=2} = \frac{13}{2} \hbar \omega \]

(b) Any number of bosons can occupy each level:

\[ E_N = N \hbar \omega_{n=0} = N \frac{1}{2} \hbar \omega, \quad N = 1, \ldots 5 \]  

(c) Compared with the case $B = 0$ when the energy did not depend on the spin orientaiton, each energy level splits into two $E \rightarrow E_n \pm \mu B$ where the spin orientation (up or down) controls the sign in this expression. For $\mu B = \hbar \omega$, $E \rightarrow \hbar \omega (n + \frac{1}{2}) \pm \hbar \omega$ The lowest single particle levels:

\[
\begin{align*}
E_0 &= \frac{1}{2} \hbar \omega - \hbar \omega = -\frac{1}{2} \hbar \omega \\
E_1 &= \frac{3}{2} \hbar \omega - \hbar \omega = \frac{1}{2} \hbar \omega \\
E_2 &= \frac{5}{2} \hbar \omega - \hbar \omega = \frac{3}{2} \hbar \omega \\
E_3 &= \frac{7}{2} \hbar \omega + \hbar \omega = \frac{5}{2} \hbar \omega \\
E_4 &= \frac{9}{2} \hbar \omega - \hbar \omega = \frac{7}{2} \hbar \omega \\
E_5 &= \frac{11}{2} \hbar \omega + \hbar \omega = \frac{9}{2} \hbar \omega
\end{align*}
\]

Each of the states can be occupied by one fermion. Then proceed as in (a): $E_{N=1} = -\frac{1}{2} \hbar \omega$, $E_{N=2} = 0$, $E_{N=3} = \frac{1}{2} \hbar \omega$, $E_{N=4} = \frac{3}{2} \hbar \omega$, $E_{N=5} = \frac{5}{2} \hbar \omega$.

8.2. In the Heisenberg picture, operators $\hat{A}$ are time dependent, $\hat{A} \rightarrow \hat{A}_H(t) = e^{\hat{H}^* t} \hat{A} e^{-\hat{H} t}$, $\hat{H}$ being the Hamiltonian.

(i) Find the position $\hat{x}_H(t)$ and momentum $\hat{p}_H(t)$ Heisenberg operators for a harmonic oscillator.

(ii) Find $\langle x \rangle_t = \langle \alpha_0 | \hat{x}_H(t) | \alpha_0 \rangle$ and $\langle p \rangle_t = \langle \alpha_0 | \hat{p}_H(t) | \alpha_0 \rangle$ where $| \alpha_0 \rangle$ is a coherent state of the oscillator.
Solution to 8.2.

(1) By definition the Heisenberg operators are

\[ \hat{x}_H(t) = e^{i \hat{H} t / \hbar} e^{-i \hat{H} t}, \quad \hat{p}_H(t) = e^{i \hat{H} t / \hbar} p e^{-i \hat{H} t} \]  

(1)

where \( \hat{H} = \frac{\hat{p}^2}{2m} + \frac{m \omega^2 \hat{x}^2}{2} \) is the harmonic oscillator Hamiltonian.

The simplest method to find the operators is to use the creation/annihilation operators, \( \hat{a} \) and \( \hat{a}^\dagger \), through which

\[ \hat{x} = \sqrt{\frac{\hbar}{2m\omega}} (\hat{a} + \hat{a}^\dagger), \quad \hat{p} = -i \sqrt{\frac{m\omega\hbar}{2}} (\hat{a} - \hat{a}^\dagger), \quad \hat{H} = \hbar \omega (\hat{a}^\dagger \hat{a} + \frac{1}{2}) \]  

(2)

so that

\[ \hat{x}_H(t) = \sqrt{\frac{\hbar}{2m\omega}} (\hat{a}_H(t) + \hat{a}_H^\dagger(t)), \quad \hat{p}_H(t) = -i \sqrt{\frac{m\omega\hbar}{2}} (\hat{a}_H(t) - \hat{a}_H^\dagger(t)) \]  

(3)

To find \( \hat{a}_H(t) \) recall the equation for Heisenberg operators:

\[ \frac{d}{dt} \hat{a}_H = \frac{i}{\hbar} [\hat{H}, \hat{a}_H] \]  

(4)

Calculating \( [\hat{H}, \hat{a}_H] \) from scratch...

\[ [\hat{H}, \hat{a}_H] = [\hat{H}, e^{i \hat{H} t / \hbar} \hat{a} e^{-i \hat{H} t / \hbar}] = e^{i \hat{H} t / \hbar} [\hat{H}, \hat{a}] e^{-i \hat{H} t / \hbar} = e^{i \hat{H} t / \hbar} [\hbar \omega (\hat{a}^\dagger \hat{a} + \frac{1}{2}), \hat{a}] e^{-i \hat{H} t / \hbar} = \hbar \omega e^{i \hat{H} t / \hbar} (\hat{a}^\dagger \hat{a} - \hat{a} \hat{a}^\dagger) e^{-i \hat{H} t / \hbar} = \hbar \omega e^{i \hat{H} t / \hbar} (\hat{a}^\dagger \hat{a} + \hat{a} \hat{a}^\dagger) e^{-i \hat{H} t / \hbar} = -i \]

Thus,

\[ \frac{d}{dt} \hat{a}_H = -i \omega \hat{a}_H, \quad \hat{a}_H(0) = \hat{a} \Rightarrow \hat{a}_H(t) = \hat{a} e^{-i \omega t} \Rightarrow \hat{a}_H^\dagger(t) = \hat{a}^\dagger e^{i \omega t} \]  

(5)

Finally, we get

\[ \hat{x}_H(t) = \sqrt{\frac{\hbar}{2m\omega}} (e^{-i \omega t} \hat{a} + e^{i \omega t} \hat{a}^\dagger), \quad \hat{p}_H(t) = -i \sqrt{\frac{m\omega\hbar}{2}} (e^{-i \omega t} \hat{a} - e^{i \omega t} \hat{a}^\dagger) \]  

(6)

(ii) Calculating \( \langle x \rangle_t = \langle \alpha_0 | \hat{x}_H(t) | \alpha_0 \rangle \) and \( \langle p \rangle_t = \langle \alpha_0 | \hat{p}_H(t) | \alpha_0 \rangle \) where \( | \alpha_0 \rangle \)...

Recall that

\[ \hat{a} | \alpha \rangle = \alpha | \alpha \rangle, \quad \langle \alpha | \hat{a}^\dagger = \alpha^* \langle \alpha | \]  

(7)

Calculating \( \langle x \rangle_t \)...

\[ \langle x \rangle_t = \langle \alpha_0 | \hat{x}_H(t) | \alpha_0 \rangle = \langle \alpha_0 | \sqrt{\frac{\hbar}{2m\omega}} (e^{-i \omega t} \hat{a} + e^{i \omega t} \hat{a}^\dagger) | \alpha_0 \rangle = \sqrt{\frac{\hbar}{2m\omega}} \left( e^{-i \omega t} \langle \alpha_0 | \hat{a} | \alpha_0 \rangle + e^{i \omega t} \langle \alpha_0 | \hat{a}^\dagger | \alpha_0 \rangle \right) = \sqrt{\frac{\hbar}{2m\omega}} \left( e^{-i \omega t} \alpha_0 | \alpha_0 \rangle + e^{i \omega t} \alpha_0^* \langle \alpha_0 | \alpha_0 \rangle \right) = \sqrt{\frac{\hbar}{2m\omega}} (e^{-i \omega t} \alpha_0 + e^{i \omega t} \alpha_0^*) \]  

Same story for \( \langle p \rangle_t \), and we get

\[ \langle x \rangle_t = \sqrt{\frac{\hbar}{2m\omega}} (e^{-i \omega t} \alpha_0 + e^{i \omega t} \alpha_0^*) \]  

(8)

\[ \langle p \rangle_t = -i \sqrt{\frac{m\omega\hbar}{2}} (e^{-i \omega t} \alpha_0 - e^{i \omega t} \alpha_0^*) \]  

(9)

Present complex number \( \alpha_0 \) as \( \alpha_0 = \alpha_{01} + i \alpha_{02} \), where \( \alpha_{01} \) and \( \alpha_{02} \) are real quantities. We observe and the initial values \( \langle x \rangle_{t=0}, \langle p \rangle_{t=0} \) are related to \( \alpha_0 \) as

\[ \langle x \rangle_0 = \sqrt{\frac{\hbar}{2m\omega}} (\alpha_0 + \alpha_0^*) = \sqrt{\frac{2\hbar}{m\omega}} \alpha_{01}, \quad \langle p \rangle_0 = -i \sqrt{\frac{m\omega\hbar}{2}} (\alpha_0 - \alpha_0^*) = \sqrt{\frac{m\omega\hbar}{2}} \alpha_{02} \]  

(10)
Expressing $\alpha_0$ in Eq. (8) via the initial values, we get

$$\langle x \rangle_t = \langle x \rangle_0 \cos \omega t + \frac{1}{m\omega} \langle p \rangle_0 \sin \omega t, \quad \langle p \rangle_t = m \frac{d\langle x \rangle_t}{dt}. \quad (11)$$

8.3. A particle is in a potential box $V(x) = \begin{cases} 0 & \text{for } |x| < a/2; \\ \infty & \text{for } |x| > a/2. \end{cases}$ At the moment $t \to -\infty$ it occupies the ground state. Time dependent potential is: $V(x,t) = -xF_0 \exp(-\frac{|x|}{a})$. In the first order of the perturbation theory, find the probability of transitions to the first excited state at $t \to +\infty$.

Solution to 8.3.

The probability of transition from the $n$'th level to the $m$'th level is given by

$$W_{nm} = \left| \frac{1}{i\hbar} \int_{-\infty}^{\infty} V_{nm}(t) e^{i\omega_{nm}t} dt \right|^2, \quad (1)$$

where $\omega_{nm} = \frac{E_n - E_m}{\hbar}$ and $V_{nm}(t) = \langle n | V(t) | m \rangle$.

To evaluate the matrix elements, use the coordinate representation, i.e., $\psi_n(x) \equiv \langle x | n \rangle$. A bit more compact calculations are when one shifts coordinates so that the potential box extends from $x = 0$ to $x = a$. Then, the boundary condition at the boundaries of the box are $\psi_n(x = 0) = 0$ together with $\psi_n(x = a) = 0$.

In the shifted coordinates, the normalized eigenstates of the unperturbed problem are

$$\langle x | n \rangle \equiv \psi_n(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}.$$

Calculating $V_{nm}(t)$:

$$V_{nm}(t) = -F_0 \exp(-\frac{|t|}{\tau}) \frac{2}{a} \int_0^a x \sin \frac{n\pi x}{a} \sin \frac{m\pi x}{a} dx =$$

$$= -F_0 \exp(-\frac{|t|}{\tau}) \frac{2}{a} \left( \frac{a}{\pi} \right)^2 \int_0^\pi y \sin ny \sin my dy =$$

$$= -F_0 \exp(-\frac{|t|}{\tau}) \frac{2}{a} \left( \frac{a}{\pi} \right)^2 \frac{1}{2} \int_0^\pi y \left( \cos (ny - my) - \cos (ny + my) \right) dy \quad (2)$$

The integral can be calculated either by integration by parts or alternatively:

$$\int_0^\pi y \cos py \, dy = \frac{p}{y^2} \int_0^\pi \sin py \, dy = \frac{1}{p^2} (-1)^p - 1.$$

Therefore the matrix element of the perturbation reads:

$$V_{nm}(t) = -F_0 \exp(-\frac{|t|}{\tau}) C_{nm} \quad (3)$$

$$C_{nm} \equiv \frac{a}{\pi^2} \left\{ \frac{1}{(n-m)^2} \left( (-1)^{n-m} - 1 \right) - \frac{1}{(n+m)^2} \left( (-1)^{n+m} - 1 \right) \right\} \quad (4)$$

In case of $n = 1$, that is the initial state is the ground state, the $C_{nm}$ can be further simplified.

$$C_{1m} = \left\{ \begin{array}{ll} \frac{4m}{\pi^2(m^2-1)^2} & \text{for } m \text{ even} \\ 0 & \text{for } m \text{ odd} \end{array} \right. \quad (5)$$

The transition amplitude from the $n$'th to the $m$'th level reads

$$a_{nm} = F_0 C_{nm} \frac{i}{\hbar} \int_{-\infty}^{\infty} \exp(-\frac{|t|}{\tau}) e^{i\omega_{nm}t} dt \quad (6)$$

The integral can be solved writing it as a sum of two integrals:

$$\int_{-\infty}^{\infty} \exp(-\frac{|t|}{\tau}) e^{i\omega_{nm}t} dt = \int_{-\infty}^{0} \exp(-\frac{t}{\tau}) e^{i\omega_{nm}t} dt + \int_{0}^{\infty} \exp(-\frac{t}{\tau}) e^{i\omega_{nm}t} dt =$$
\[
= \int_0^\infty e^{-\frac{t}{\tau}} (e^{i\omega_{nm}t} + e^{-i\omega_{nm}t}) \, dt = \\
= \frac{2\tau}{1 + (\tau\omega_{nm})^2}
\]  
\[\tag{7}\]

The probability of transition from the \(n'\)th to the \(m'\)th level reads
\[W_{nm} = \frac{F_0^2}{\hbar^2} \frac{4\tau^2}{(1 + (\tau\omega_{nm})^2)^2} |C_{nm}|^2 \]
\[\tag{8}\]

Specially for the system initially in the ground state \(n = 1\) we get:
\[W_{1m} = \begin{cases} 
\frac{F_0^2}{\hbar^2} \frac{4\tau^2}{(1 + (\tau\omega_{1m})^2)^2} \frac{(4am)^2}{\pi (m^2 - 1)^2} & \text{for } m \text{ even} \\
0 & \text{for } m \text{ odd}
\end{cases} \]
\[\tag{9}\]

**8.4.** A hydrogen atom is placed in a time-dependent homogeneous electric field, \(E(t) = E_0 e^{-t/\tau}\), where \(E_0\) and \(\tau\) are constants. At \(t = 0\) the atom is in its ground state. Calculate the probability that it will be in a 2p state at \(t \to \infty\) considering the electric field as a weak perturbation.

**Solution to 8.4.**

The full Hamiltonian \(\hat{H}\)
\[\hat{H} = \hat{H}_{\text{hydrogen}} + (-e)\varphi(r), \quad \varphi(r) = -r \cdot E(t)\]

where \(\hat{H}_{\text{hydrogen}}\) is the Hamiltonian corresponding to electron in the hydrogen atom and \(\varphi(r)\) is the scalar potential corresponding to the electric field \(E(t)\) (in the gauge where the vector potential \(A = 0\)). The perturbation to the hydrogen Hamiltonian is
\[\hat{H}_1 = (-e)\varphi(r) = er \cdot E(t)\]

Choose the \(z\)-axis in the direction of the electric field. Then,
\[\hat{H}_1(t) = ezE(t), \quad E(t) = E_0 e^{-t/\tau}\]

Applying first order perturbation theory, the amplitude of the transition from 1s to the state 2p, \(m, m = 0, \pm 1\)
\[c_{1s \rightarrow 2p,m} = \frac{i}{\hbar} \int_0^\infty \langle 2p,m | \hat{H}_1(t) | 1s \rangle e^{i\omega t} \, dt \]
\[\tag{1}\]

where \(\omega = \frac{1}{\hbar} (E_{2p} - E_{1s}) = \frac{3}{8} \frac{e^2}{\hbar a}\).

Using the coordinate representation for the matrix element,
\[c_{1s \rightarrow 2p,m} = \frac{ie}{\hbar} \int_0^\infty dt \, E(t)e^{i\omega t} \int d^3r \, \psi_{2p,m}^*(r) z \, \psi_{1s}(r) \]
\[\tag{2}\]

Using the following expressions for the hydrogen wave functions in the spherical coordinates and \(z = r \cos \theta\):
\[\psi_{2p,m} \equiv \psi_{21m} = R_{21}(r)Y_1^m(\theta, \phi) \quad ; \quad \psi_{1s} \equiv \psi_{100} = R_{10}(r)Y_0^0(\theta, \phi)\]
\[\tag{3}\]
\[Y_1^m = \begin{cases} 
- \sqrt{\frac{3\pi}{8\pi}} \sin \theta e^{i\phi} & \text{for } m \text{ even} \\
\sqrt{\frac{3\pi}{8\pi}} \cos \theta & \text{for } m \text{ odd}
\end{cases} \]
\[\tag{4}\]
\[R_{10}(r) = 2a^{-3/2}e^{-r/\alpha} \quad ; \quad R_{21}(r) = \frac{1}{24} a^{-3/2} \frac{T_e^{-r/2a}}{\alpha \infty} \]
\[\tag{5}\]
\[\int d^3r \, \psi_{2p,m}^*(r) z \, \psi_{1s}(r) = \int d\Omega Y_1^m r \psi_{21m}^* r \psi_{100} \cos \theta \cdot \int_0^{R_{21}(r)R_{10}(r)r^2} dr = \]
\[ \frac{1}{\sqrt{3}} \delta_{m,0} \frac{1}{\sqrt{6} a^4} \int_0^\infty r^4 e^{-\frac{1}{2} \frac{m^2}{a^2}} dr = \]
\[ = \frac{2^8}{3^6 \sqrt{2}} a \delta_{m,0} \quad (6) \]

The transition amplitude Eq. (2) reads now:
\[ c_{1s \rightarrow 2p,m} = \frac{2^8}{3^6 \sqrt{2}} a \delta_{m,0} \frac{ie E_0}{\hbar} \int_0^\infty dt \ e^{-\frac{t}{\hbar}} e^{i \omega t} \]
\[ \frac{1 - 1}{1 - i \omega} \quad (7) \]

The probability of transition from the 1s to a 2p level is the sum of probabilities of transition to the states with \( m = 0, \pm 1 \). Actually, the transition occurs only to the state with \( m = 0 \).
\[ P_{1s \rightarrow 2p} = |c_{1s \rightarrow 2p,m=0}|^2 = \frac{2^{16} e^2 E_0^2 a^2}{3^16 \hbar^2 \tau^2} \frac{\tau^2}{1 + \omega^2 \tau^2} \quad (8) \]

8.5. Atoms emit photons as a result of the electronic transition from the state 2p to the state 1s (2p \( \rightarrow \) 1s). The initial 2p state is one of
(i) \( l = 1, m = 1 \)
(ii) \( l = 1, m = 0 \)
(iii) \( l = 1, m = -1 \),
where as usual \( m \) is the magnetic quantum number (\( L_z = m \hbar \) is the \( z \)-projection of the angular momentum). A detector registers photons propagating in (a) \( x \), or (b) \( y \), or (c) \( z \) direction. The detector is polarization sensitive: one chooses to register either one of the two possible orthogonal linear polarizations or one of the two circular polarizations.

For each of the directions of propagation and the selected polarizations, find whether the detector registers photons.

Hint: Use the dipole approximation. Since the atom emit photons with the polarization vector \( \boldsymbol{\epsilon} \) provided the dipole matrix element \( \langle n_f, l_f, m_f | \hat{r} \cdot \boldsymbol{\epsilon}^* | n_i, l_i, m_i \rangle \) is not equal to zero, the polarization sensitive detector registers photons with polarization \( \boldsymbol{\epsilon} \) only if the corresponding matrix element is not zero.

For definiteness, choose the linear polarization along \( x, y \) or \( z \) axes. Don't forget that the polarization vector \( \boldsymbol{\epsilon} \) is orthogonal to the direction of propagation.

Solution to 8.5.

An arbitrary polarization vector \( \boldsymbol{\epsilon} \) can be presented as a sum
\[ \boldsymbol{\epsilon} = \epsilon_+^{(z)} \epsilon_+^{(z)} + \epsilon_-^{(z)} \epsilon_-^{(z)} + \epsilon_z \epsilon_z \quad \text{where} \quad \epsilon_\pm = \epsilon_z \pm i \epsilon_y \]
and \( \epsilon_{x,y,z} \) are unit vectors in the corresponding direction. Given polarization vector \( \boldsymbol{\epsilon} \), the components \( \epsilon_\pm^{(z)} \) and \( \epsilon_z \) can be found as
\[ \epsilon_\pm^{(z)} = \frac{1}{2} \epsilon \cdot \epsilon_\pm^{(z)} \quad \epsilon_z = \epsilon \cdot \epsilon_z. \quad (1) \]

The combination \( \boldsymbol{\epsilon}^* \cdot \boldsymbol{r} \) in the dipole matrix element can be presented as
\[ \epsilon^* \cdot \boldsymbol{r} = \epsilon_+^{(z)} x_+ + \epsilon_-^{(z)} x_- + \epsilon_z^* z \quad \text{where} \quad x_{\pm} = x \pm iy \]

For the initial state \( |l = 1, m \rangle \) and emission of the photon with polarization \( \boldsymbol{\epsilon} \), the dipole matrix element is
\[ \langle 0, 0 | \epsilon^* \cdot \boldsymbol{r} | l = 1, m \rangle = \epsilon_+^{(z)}\langle 0, 0 | x_+ | 1, m \rangle + \epsilon_-^{(z)}\langle 0, 0 | x_- | 1, m \rangle + \epsilon_z^* \langle 0, 0 | z | 1, m \rangle. \]
Note now that \( x_{\pm} = r e^{\pm iy} \) and after the integration with respect to angles in the matrix elements one sees that:
The first term (with \( x_- \)) is not zero only if \( m = +1 \), the second is not zero only if \( m = -1 \), and the last one if \( m = 0 \). This means that when atom initially in the state with \( m = 1 \), it emits only photons whose
polarization has finite component component $e^{(z)}_+$. If $m = -1$, the polarization must have a finite component $e^{(z)}_-$, and when $m = 0$, the polarization vector must have $z-$component.

Answering the question...

**Detector which registers photons propagation in $z$-direction.**
Possible polarization $e$: linear in $y$ and $z$ directions, circular right/left $(e_y + / - ie_z)$.
Initial state $m = 1$, polarization must have $e^{(z)}_+$ component found from Eq. (1), *i.e.*, the detector which selects polarization $e$ starts clicking only of $e \cdot (e_y - ie_y) \neq 0$.
The component is finite for $y$-polarization and both circular polarizations but not for linear $z-$polarization
Initial state $m = -1$, polarization must have finite $e^{(z)}_-$-component.
The component is finite for $y$-polarization and both circular polarizations but not for linear $z-$polarization
Initial state $m = 0$, polarization must have overlap with $e_z$.
The component is finite for $z$-polarization and both circular polarizations but not for linear $y-$polarization.

**Detector which registers photons propagation in $y$-direction.**
Possible polarization $e$: linear in $x$ and $z$ directions, circular right/left $(e_x + / - ie_z)$.
Initial state $m = 1$, polarization must have $e^{(z)}_+$ component found from Eq. (1).
The component is finite for $x$-polarization and both circular polarizations but not for linear $z-$polarization
Initial state $m = -1$, polarization must have overlap with $e^{(z)}_-$.
The component is finite for $x$-polarization and both circular polarizations but not for linear $z-$polarization
Initial state $m = 0$, polarization must have overlap with $e_z$.
The component is finite for linear $z$-polarization and both circular polarizations but not for linear $x-$polarization.

**Detector which registers photons propagation in $z$-direction.**
Possible polarization $e$: linear in $x$ and $y$ directions, circular right/left $(e_x + / - ie_y)$.
Initial state $m = 1$, polarization must have $e^{(z)}_+$ component found from Eq. (1).
The component is finite for both $x$- and $y$- linear polarization and for right circular polarizations but not for left
Initial state $m = -1$, polarization must have overlap with $e^{(z)}_-$.
The component is finite for $x$- and $y$- linear polarization and left circular but not for right circular polarization
Initial state $m = 0$, polarization must contain component $e_z$. This is not possible for this direction of propagation.
Therefore, no photon will be detected.